



## Problem A

### Minimum Factorial as a Multiple

(Time Limit: 1 second)

Let  $k$  be a positive integer. The factorial of  $k$ , denoted by  $k!$ , is equal to  $1 \times 2 \times \dots \times k$ . Given an input number  $n$ , we want to find the smallest positive integer  $k$  such that  $k!$  is a multiple of  $n$ .

#### Input Format

The input begins with a single positive integer  $m$ , which specifies the number of test cases. Then, each of the following  $m$  lines corresponds to a test case, which contains a single positive integer  $n$ .

#### Output Format

For each test case, output, on a separate line, the smallest positive integer  $k$  such that its factorial  $k!$  is a multiple of  $n$ . (Note: For all test cases, it is guaranteed that the corresponding output  $k$  would be at most 12.)

#### Technical Specification

- $1 \leq m \leq 100$
- $1 \leq n \leq 10^7$
- For each integer  $n$  in the input test cases, there exists some  $k$  with  $1 \leq k \leq 12$  such that  $k!$  is a multiple of  $n$

#### Example

Sample Input:	Sample Output:
3	1
1	5
40	12
136080	



## Problem B

### Population Count

(Time Limit: 1 second)

In Binary City, there are numerous houses. The city government numbered the houses from 1 to  $n$ . Amazingly, house  $k$  can accommodate up to  $c(k)$  persons where  $c(k)$  is the number of 1's in the binary representation of  $k$ . For example, house 113 can accommodate 4 persons, since 113 in decimal is  $1110001_2$  in binary numeral system.

After the mayor of Binary City proposed his “Make Big Money!” project, numerous people moved to Binary City, since they want to become the richest people in the world. Now, all houses in Binary City are full of people.

The civil affairs director of Binary City asks you to help the city government to do some statistics for answering inquiries from the city council. Each inquiry consists of two integers  $b$  and  $e$ , and your task is to compute the population living in houses  $b, b + 1, \dots, e - 1, e$  which is  $\sum_{k=b}^e c(k)$ .

#### Input Format

The first line of the input contains an integer  $m$  indicating the number of inquiries. Each of the following lines is an inquiry, and there are two numbers  $b$  and  $e$  separated by blanks.

#### Output Format

For each inquiry, output the population living in houses  $b, b + 1, \dots, e - 1, e$  on one line.

#### Technical Specification

- $1 \leq m \leq 20$ ,  $1 \leq b \leq e \leq 10^4$ , and  $e \leq n$ .

#### Example

Sample Input:	Sample Output:
3	17



1 10	4
113 113	64613
1 10000	

## Problem C

### Boxes

(Time Limit: 2 seconds)

You have  $n$  boxes in a line on the table numbered  $1 \dots n$  from left to right. Your task is to simulate 4 kinds of commands:

- (i) 1  $X$   $Y$ : move box  $X$  to the left to  $Y$  (ignore this if  $X$  is already the left of  $Y$ )
- (ii) 2  $X$   $Y$ : move box  $X$  to the right to  $Y$  (ignore this if  $X$  is already the right of  $Y$ )
- (iii) 3  $X$   $Y$ : swap box  $X$  and  $Y$
- (iv) 4: reverse the whole line.

Commands are guaranteed to be valid, i.e.  $X$  will be not equal to  $Y$ .

For example, if  $n = 6$ , after executing 1 1 4, the line becomes 2 3 1 4 5 6. Then after executing 2 3 5, the line becomes 2 1 4 5 3 6. Then after executing 3 1 6, the line becomes 2 6 4 5 3 1. Then after executing 4, then line becomes 1 3 5 4 6 2.

### Input Format

The input contains several test cases and is terminated by End-Of-File (EOF). Each test case begins with a line containing 2 integers  $n, m$ . Each of the following  $m$  lines contain a command.

### Output Format

For each test case, print the sum of numbers at odd-indexed positions. Positions are numbered 1 to  $n$  from left to right.

### Technical Specification

- For each test case, you may assume that  $n, m \leq 100000$ .
- The sum of  $n$ 's and  $m$ 's over all test cases is at most 524288.



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**Example**

Sample Input:	Sample Output:
6 4	12
1 1 4	9
2 3 5	2500050000
3 1 6	
4	
6 3	
1 1 4	
2 3 5	
3 1 6	
100000 1	
4	



## Problem D

### RSSI fingerprinting

(Time Limit: 1 second)

You are an engineer of a tech company that builds indoor positioning systems. You get a set of RSSI (Received Signal Strength Indication) measurements measured by your colleague. RSSI is an indication of the power level being received by the receive radio after the antenna and possible cable loss. Therefore, the higher the RSSI value, the stronger the signal. Your task is to delete the stale RSSI measurements and to transform the remaining ones into a specific fingerprinting format defined by your company.

#### Input Format

The first line of the input contains an integer  $n$ ,  $0 \leq n \leq 10$ , indicating the number of test cases. For each test case, the first line contains an integer  $m$ ,  $0 \leq m \leq 99$ , indicating the number of RSSI measurements reported from an RSSI measurement tool. Each of the following  $m$  lines contains an RSSI measurement and each measurement contains one string  $S$  and two integers  $r$  ( $-99 \leq r \leq -20$ ) and  $t$  ( $0 \leq t$ ). The string  $S$  is the MAC address of a Wi-Fi access point. A MAC address is a 12 digits (6-bytes) of hexadecimal numbers written in the format MM:MM:MM:MM:MM:MM (e.g., B0:C1:9E:24:12:9D). Note that all the MAC addresses within the same test case are distinct. The integer  $r$  is the measured RSSI value and the integer  $t$  is the last time stamp the RSSI measurement tool detecting the access point. If  $t$  exceeds 1000, this measurement is stale and you must drop it.

#### Output Format

For each test case, output at most 3 RSSI measurements with last time stamp not exceeding 1000 in decreasing order based on the RSSI values. The qualified RSSI measurements with the same RSSI values are outputted according to the lexicographic order of their MAC addresses. For each RSSI measurement, we output its fingerprinting format, which contains its MAC address and the RSSI value separated by a blank, in a line.



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Example

Sample Input:	Sample Output:
2	E8:CC:18:F2:4A:48 -71
5	BC:99:11:62:23:E7 -83
78:44:76:F3:95:BE -43 1380	F8:35:DD:B8:CE:92 -83
80:1F:02:BA:90:3E -85 532	FA:45:DD:B8:FE:72 -63
E8:CC:18:F2:4A:48 -71 336	
F8:35:DD:B8:CE:92 -83 972	
BC:99:11:62:23:E7 -83 528	
2	
E8:CC:18:F2:4A:48 -81 1336	
FA:45:DD:B8:FE:72 -63 972	



## Problem E

# Generalized Pascal's Triangle

(Time Limit: 2 seconds)

Pascal's triangle is a triangular array in which each number can be calculated by the sum of the two numbers directly above that number as shown in Figure 1. One of its prominent applications is to determine the coefficients which arise in binomial expansion (or binomial theorem), say the coefficients of  $(x+y)^n$ . Figure 1 also illustrates how to derive the elements layer by layer and shows that layer  $k$  gives the coefficients of the expansion of  $(x+y)^k$ . We now want to generalize Pascal's triangle in higher dimension and consider the three-dimensional version. This three-dimensional version can be associated with the coefficients of the trinomial expansion,  $(x+y+z)^n$ , and its shape is a tetrahedron as shown in Figure 2(a) instead of a triangle. Figure 2(b)-(e) present the elements on layer 0, 1, 2, 3, and 4 (center) as well as demonstrate the relation between layer  $i$  and layer  $i+1$  (right). The element on layer  $i$  can be derived from two or three elements on layer  $i-1$ . This is the same as the coefficients in the expansion of trinomial  $(x+y+z)^i$  (on layer  $i$ ) that can be calculated from the coefficients of  $(x+y+z)^{i-1}$  (on layer  $i-1$ ) with the sum of two or three numbers. Now, given an integer  $n$  as the layer number, please list all the elements on layer  $n$  in a triangular array.

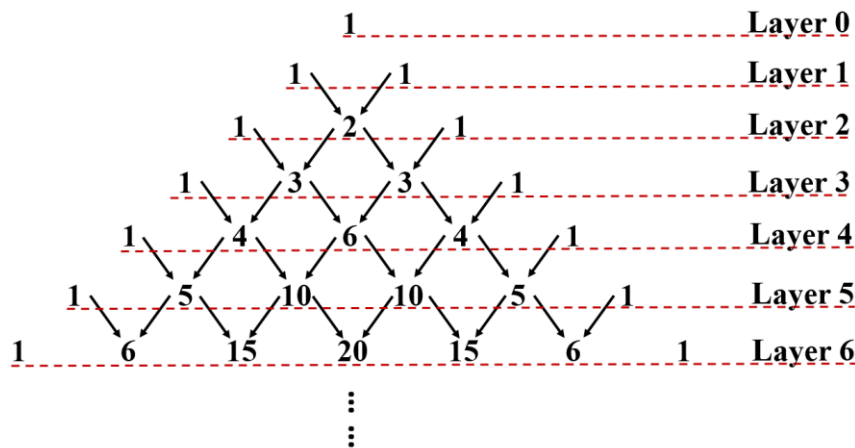
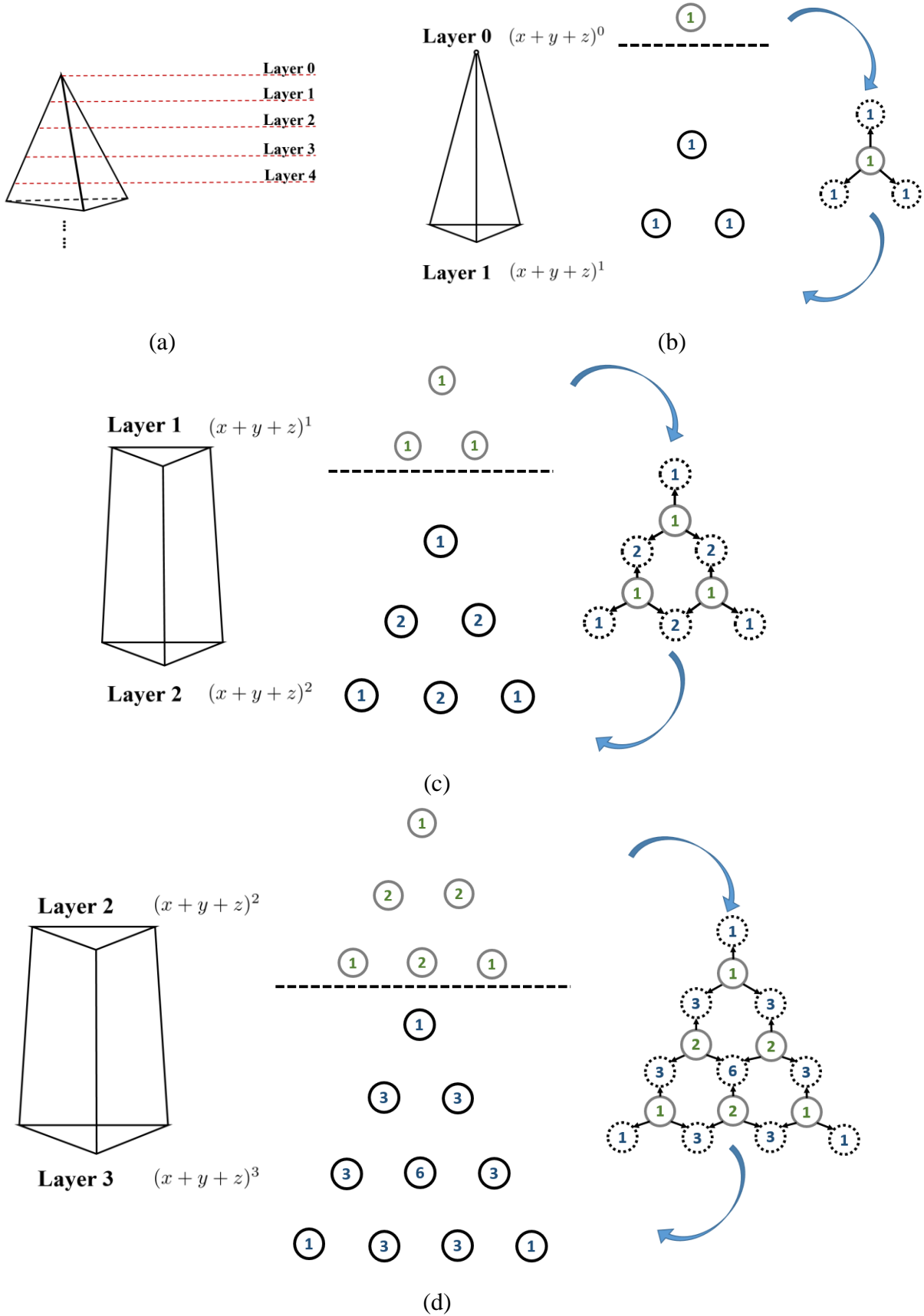


Figure 1. Pascal's triangle

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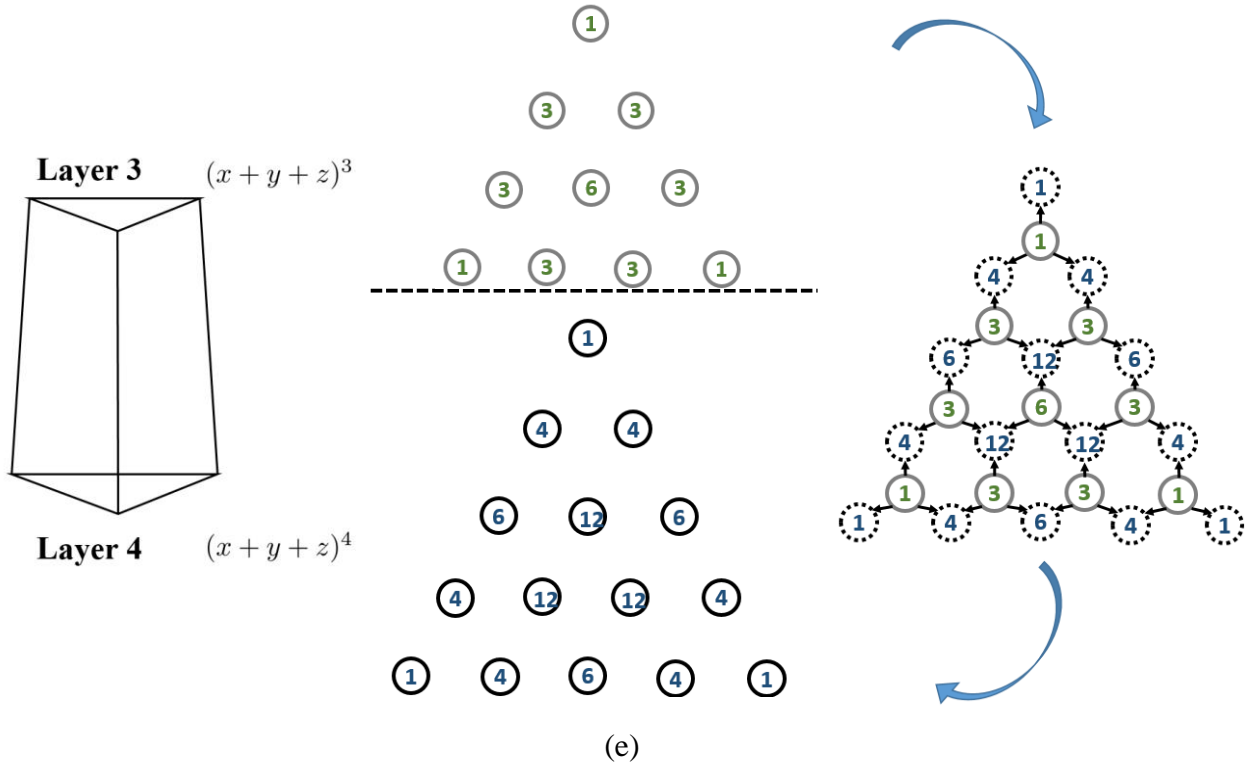


Figure 2. Three-dimensional version of Pascal's triangle: (a) The shape and (b)-(e) presenting the elements on layer 0, 1, 2, 3, and 4 (center) as well as demonstrating the relation between layer  $i$  and layer  $i+1$  (right) with associated trinomial expansion

### Input Format

The input contains several test cases and is terminated by End-Of-File (EOF). Each test case is an integer  $n$ .

### Output Format

For each test case, the output is like a triangular array and shall be denoted in  $n+1$  lines depending on the input  $n$ . The first line has one element (as the first coefficient in the expansion of  $(x+y+z)^n$ ) and the second line has two elements and etc. The last line has  $n+1$  elements that are the coefficients of the binomial expansion of  $(x+y)^n$ . The first column of the output array thus has  $n+1$  elements and those elements are also the coefficients of the binomial expansion of  $(x+y)^n$ . In each line of the output, all elements are separated by space key as delimiter.

### Technical Specification

- $0 \leq n \leq 20$



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**Example**

Sample Input:	Sample Output:
2	1
3	2 2
4	1 2 1
	1
	3 3
	3 6 3
	1 3 3 1
	1
	4 4
	6 12 6
	4 12 12 4
	1 4 6 4 1



## Problem F

### Sequence Decoding

(Time Limit: 1 second)

The amino acids in proteins are classified into two types of elements, hydrophobic (nonpolar) and hydrophilic (polar). Hydrophobic and hydrophilic are denoted by H and P respectively. A protein is represented by a sequence of H and P such as PPHHHPHPH. In order to reduce the representation length of a sequence, we use the notation  $k[S]$  to denote the repeated sequence of  $k$  times sequence  $S$ , where  $2 \leq k \leq 9$ . A *legal sequence* is defined as following.

- A sequence consists of only one character 'H' or 'P' is a legal sequence.
- Let  $S_1$  and  $S_2$  be legal sequences. Then the sequence concatenated by  $S_1$  and  $S_2$  is also a legal sequence.
- Let  $S$  be a legal sequence. Then the sequence  $k[S]$  is also a legal sequence, where  $2 \leq k \leq 9$ .

For example, PHPHPHPH is encoded as  $4[PH]$ . Note that a repeated sequence may contains repeated sequences recursively such as  $2[PH4[P]4[H]]$ .

Given a nonempty encoded protein sequence  $S$ , your job is to expand  $S$  to its original sequence. That is, you should expand  $2[PH4[P]4[H]]$  to PHPPPPHHHHPHPPPPHHHH.

### Input Format

The first line is an integer  $n$  indicating the number of test cases. Each of the next  $n$  lines consists of a legal sequence composed by number digits '2'~'9', '[', ']', 'P' and 'H'.

### Output Format

For each test case, output the expanding sequence in one line.

### Technical Specification

- $1 \leq n \leq 10$ .
- All the inputs are legal.
- The length of each input sequence is less than 50.
- The length of each expanded sequence is less than 1000.



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**Example**

Sample Input:	Sample Output:
3 PHPHP 2 [ 3 [ P ] H 2 [ P ] ] HH 2 [ P 3 [ H ] ] P	PHPHP PPPHPPPPHPP HHPHHHPHHP



## Problem G

### Cycles

(Time Limit: 1 second)

The ACM kingdom has  $n \geq 2$  cities, numbered from 1 to  $n$ , and  $m$  (bidirectional) roads. Each road connects two distinct cities. For all  $i, j \in \{1, 2, \dots, n\}$ , there is at most one road connecting cities  $i$  and  $j$ . We want to know whether both conditions below hold:

- The ACM kingdom is connected. That is, city  $i$  is reachable from city  $j$  via a finite number of roads for all  $i, j \in \{1, 2, \dots, n\}$ .
- The ACM kingdom has at least two distinct cycles. Here a cycle starts at a city, goes through a positive number of roads and finally ends at the starting city. Except for the starting city, no city can appear twice on a cycle. Furthermore, two cycles are said to be distinct as long as their sets of roads are not exactly the same.

If the two conditions above hold, we would like to further know whether the ACM kingdom has at least three distinct cycles.

### Input Format

The first line is the number of test cases. The first two lines of each test case are  $n$  and  $m$ , in that order. Then each of the  $m$  roads is specified by the two cities (separated by a space) that it connects.

### Output Format

Each test case requires the following output: If it is not true that the ACM kingdom is connected and has at least two distinct cycles, output “n” in a line. Otherwise, output “y” in a line if the ACM kingdom has exactly two distinct cycles and “y: there are at least three cycles” in a line if the ACM kingdom has at least three distinct cycles.

### Technical Specification

- There are at most 15 test cases.
- $2 \leq n \leq 9$ .



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Example

Sample Input:	Sample Output:
3	y
9	y: there are at least three cycles
10	n
1 2	
1 3	
3 5	
5 6	
5 7	
7 3	
2 4	
9 4	
4 8	
2 8	
6	
7	
4 3	
4 5	
4 6	
1 4	
2 4	
3 5	
5 6	
3	
2	
1 2	
3 2	



## Problem H

### Countable Rational Numbers

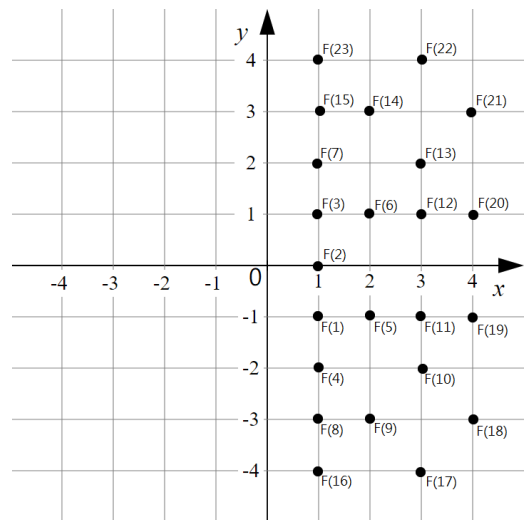
(Time Limit: 3 seconds)

The set of rational numbers is described as  $Q = \left\{ \frac{y}{x} \mid x, y \in Z, x > 0, \gcd(x, y) = 1 \right\}$  where  $Z$  is the set of integers and  $\gcd(x, y)$  is the greatest common divisor of  $x$  and  $y$ . For examples,  $\frac{1}{2}, \frac{-1}{2}, \frac{3}{1}, \frac{-3}{1}, \dots$  are rational numbers while  $\frac{2}{4}, \frac{1}{-2}, \frac{8}{2}, \frac{-3}{-2}, \dots$  are syntactically not rational numbers since they violate the constraints  $x > 0$  or  $\gcd(x, y) = 1$ . It is well-known that the cardinality of rational numbers is countable infinity. However how to prove that is to find a bijection from natural numbers  $N$  to rational numbers  $Q$ .

Consider two rational numbers,  $\frac{y_1}{x_1}$  and  $\frac{y_2}{x_2}$ . Let  $m_1 = \max(x_1, \text{abs}(y_1))$ ,  $m_2 = \max(x_2, \text{abs}(y_2))$ . We define  $\frac{y_1}{x_1} \triangleleft \frac{y_2}{x_2}$  if one of following constraints is satisfied.

1.  $m_1 < m_2$
2.  $m_1 = m_2$  and  $y_1 < y_2$
3.  $m_1 = m_2$  and  $y_1 = y_2$  and  $x_1 \times y_1 > x_2 \times y_2$

According to the ordering  $\triangleleft$ , we can build a bijection  $F$  from  $N$  to  $Q$  by enumerating elements of  $Q$  from the smallest to larger as  $\frac{-1}{1} \triangleleft \frac{0}{1} \triangleleft \frac{1}{1} \triangleleft \frac{-2}{1} \triangleleft \frac{-1}{2} \triangleleft \frac{1}{2} \triangleleft \frac{2}{1} \triangleleft \frac{-3}{1} \triangleleft \frac{-3}{2} \triangleleft \frac{-2}{3} \triangleleft \frac{-1}{3} \triangleleft \frac{1}{3} \triangleleft \frac{2}{3} \triangleleft \frac{3}{2} \triangleleft \frac{3}{1} \dots$ , that is  $F(1) = \frac{-1}{1}, F(2) = \frac{0}{1}, F(3) = \frac{1}{1}, F(4) = \frac{-2}{1}, \dots$ . For each rational number



$F(n) = \frac{y}{x}$ , we can plot it at  $(x, y)$  on  $x - y$  plane as the above picture. On the picture, we can see that there is no rational numbers on the left part since the value of  $x$  has to be larger than zero. We can also understand that there is no rational number at  $(4, 2)$  since  $\frac{2}{4}$  is not a rational number because  $\gcd(4, 2) = 2 \neq 1$ .



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For any given integers  $y$  and  $x$ , can you find an integer  $n$  such that  $F(n) = \frac{y}{x}$ .

### Input Format

The first line is a number indicating the number of test cases. Each test case has two integers  $y$  and  $x$ . (Remark: "+" and "-0" do not appear in test cases.)

### Output Format

For each test case, please output  $n$  if there exists an integer  $n$  such that  $F(n) = \frac{y}{x}$  and please output 0 if  $\frac{y}{x}$  is syntactically not a rational number.

### Technical Specification

- There are at most 10 test cases.
- $-1,000,000 < x < 1,000,000$
- $-1,000,000 < y < 1,000,000$

### Example

Sample Input:	Sample Output:
5	0
4 2	11
-1 3	13
2 3	0
1 -3	23
4 1	